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Explosive energy release in magnetic shocks

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We show that a magnetic shock whose initial density and/or magnetic perturbation exceeds the Hugoniot limit may lead to substantial and rapid energy release in low β plasmas (such as occur in the magnetospheres of neutron stars). We illustrate this effect for a fast Magnetohydrodynamic perturbation, as well as for large density perturbations which can be naturally created in low β plasmas. Using the Riemann solution and simulations, we show that slow modes of finite magnitudes and Alfvénic perturbations can generate strong density perturbations. These perturbations develop into shocks, resulting in efficient energy release.

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Efficient magnetic field dissipation is an important and complicated issue in astrophysics. That is, it is often observed that dissipation is "fast," i.e., becomes independent of the local "molecular" diffusion coefficients, and so proceeds on time scales much faster than the Ohmic decay times predicted on the basis of the high conductivity of cosmic plasmas; in many cases, this time scale can become comparable to the Alfvén time [1]. Shock waves of finite amplitude are known to be efficient in this respect. This is especially the case for low-β plasmas ($\beta \equiv$gas pressure/magnetic pressure); in that case, magnetic field dissipation can provide a major contribution to the heating. However, magnetic energy release seems to be limited by the fact that the magnetic jump (as well as the density jump) in a shock is bounded from above by the Hugoniot adiabat (viz., [2–5]), that is, it cannot exceed $\alpha=(\gamma+1)/(\gamma-1)$. The question, therefore, is what kind of energy release can occur in a system with initial density or/and magnetic field perturbations which exceed α? Such shocks can arise naturally in the context of accretion onto the magnetosphere of a neutron star, in which magnetic Rayleigh-Taylor instability can generate highly overdense "blobs" of magnetized matter which fall to the surface of a neutron star at the free-fall speed ($\sim c$); in such cases, $\beta \ll 1$, and nonweak magnetic and density perturbations may easily appear [6]. In addition, as we shall show, density perturbations exceeding α naturally appear in low-β plasmas, even for magnetic perturbations with modest magnetic Mach numbers, $M_A \sim 1$. In particular, slow magnetic perturbations correspond to strong density variations. Furthermore, Alfvén waves of finite amplitudes can give rise to rather strong density fluctuations [7]; the cited literature considers weak nonlinear perturbations (and nearly parallel propagation). As a consequence, entropy is nearly conserved in these systems. As nonadiabatic energy release is the main issue we are concerned with, we reconsider this problem for moderately strong perturbations, and provide numerical simulations for such perturbations using the full set of resistive (collisional) single-fluid MHD equations (viz., equations (66.1-3) in [2]).

To begin with, we note that initial magnetic fluctuations exceeding α do not result in shock waves with jumps exceeding this value, this being excluded by the Hugoniot adiabatic. The question is then when and how this extra energy (of the magnetic field) is released. We show here that this energy release can happen even before any significant energy dissipation in a shock takes place.

There are a number of methods available for treating hydrodynamic and magnetohydrodynamic shocks [4]. Here, we are especially concerned with preserving the correct density evolution: We know that the density evolution equation is not dissipative, while most numerical methods for treating shocks are inherently dissipative. In order to minimize this type of dissipation, and in order to avoid numerical instabilities, we followed a prescription previously used to evolve magnetic fields in such a way as to exactly preserve the magnetic topology [8,9].

The magnetic Hugoniot adiabat can be written as [2]

$$P_2=P_1+\frac{\{P_1\tilde{\alpha}+(\mathbf{B}_{t2}-\mathbf{B}_{t1})^2/(8\pi)\}(\zeta-1)}{\alpha-\zeta}, \quad (1)$$

where $\tilde{\alpha}=\alpha+1$, subscript 1 corresponds to the medium ahead of the shock, subscript 2 refers to the medium after the shock, and $\zeta=\rho_2/\rho_1>1$ [10]. Consider first a perpendicular shock, $B_x=0$, i.e., a shock propagating in the x direction; then $B_{t2}/B_{t1}=\zeta$. We also consider $\beta_1=P_1/(B_1^2/8\pi)\ll 1$, i.e., a low-$\beta$ ambient plasma. In that case, the magnetic pressure starts to compete with P_1 when $\zeta-1 \geqslant (\beta_1\tilde{\alpha}\gamma)^{1/2}$. A higher threshold for the level of perturbation is obtained if $\zeta-1 \geqslant (\beta_1\tilde{\alpha}/\gamma)^{1/3}$, in which case the temperature increment of the plasma due to the shock wave heating is comparable to the temperature itself, corresponding to quite substantial heating (in spite of still relatively low compression, $\zeta-1 \ll 1$). If, on the other hand, $\zeta-1 \geqslant 1$, i.e., the magnetic Mach number $M_A=v_x/v_A \gtrsim 1$ (v_A, the Alfvén velocity), then the magnetic term is substantially bigger than the pressure. In particular, if $\gamma=\frac{5}{3}$, the case we consider below, then $\alpha=4$, and for $\zeta-1 \geqslant 2.06$, $\beta_2=P_2/(B_2^2/8\pi)\geqslant 1$. In that case, the heating is so intense that the plasma conditions change from ambient $\beta_1 \ll 1$ to an aftershock value of $\beta_2>1$. Finally, the most striking features appear when the initial perturbation exceeds the limit $\zeta-1 \geqslant \alpha-1$, which, according to Eq. (1), corresponds to a singularity in the adiabat. Therefore, we anticipate that at the initial stages, the perturbation is going to develop a shock-front structure, as always when diffusion

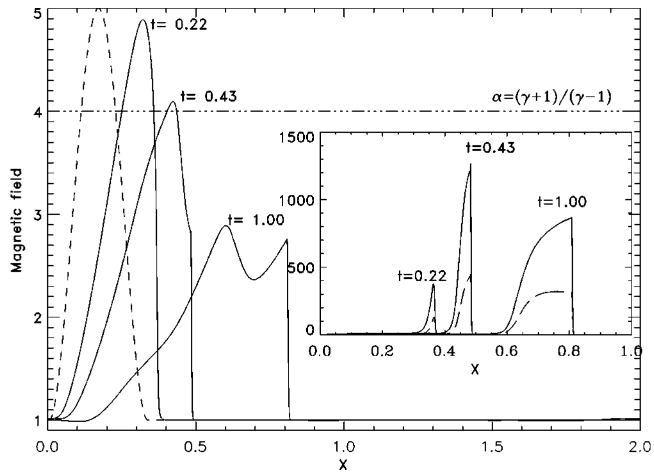

FIG. 1. Magnetic field evolution as a function of time. The initial spatial magnetic field distribution is depicted with a dashed line. Time is measured in terms of the Alfvén time, $t_A = \ell / v_A$, ℓ is the characteristic scale of the initial perturbation, and v_A is the maximal initial Alfvén velocity. The inset depicts the internal energy density, $\rho(x,t)\varepsilon(x,t)/(\rho_1 \varepsilon_1)$ (solid line), and the normalized temperature distribution, $T(x,t)/T_1$ (dashed line).

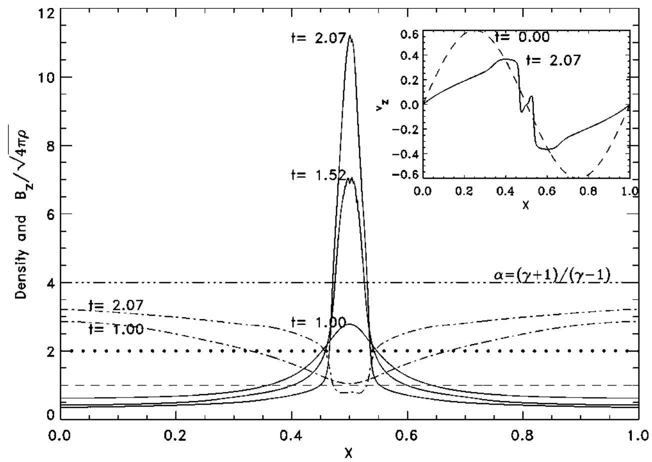

FIG. 2. Evolution of the spatial density distribution as a function of time (again in terms of t_A); the initial spatial distribution is depicted with a dashed line, and subsequent density distributions are depicted by solid lines. The evolution of the Alfvén velocity is depicted with a dotted line (initially) and with dashed-dotted lines for later times. The inset shows the evolution of v_z, which essentially generates the density gradient and any other quantities from initial constants.

can be neglected. After forming a discontinuity, the energy of the perturbation will decrease dramatically, so that the density jump is no longer larger than the limiting value, α. This stage of the process is accompanied by very fast energy dissipation, i.e., in a time shorter than the Alfvén time.

In order to verify this statement, we have simulated a perpendicular shock ($B_x = 0$) with $\beta_1 = 0.01$, and initial magnetic field perturbation with amplitude exceeding α (Fig. 1). To be more specific, initial perturbations are constructed to correspond to the Riemann solution, e.g., given initial $B_z(x)$, we have at $t=0$, $\rho = \rho_1 B_z / B_1$, $v_x = 2 v_{A1}[(\rho/\rho_1)^{1/2} - 1]$, so that initial $M_A \approx 2$ (see, e.g., [2,5]). The Riemann solution then leads to the formation of a shock front, independent of the initial amplitude. However, as $B_2/B_1 > \alpha$, a portion of the plasma behind the front is extensively heated, mainly due to sharp magnetic gradients (Fig. 1), i.e., due to the Ohmic decay term $\sim (\nabla \times \mathbf{B})^2$ in the energy equation (66.3) of [2]. Already at $t = 0.22 t_A$, where t_A is the Alfvén time (the shortest time scale in the problem), the magnetic energy (both locally and globally) has decreased, so that—as seen from the inset of the figure—the temperature has increased 127.4-fold ($T_2/T_1 = 127.4$), while $P_2/P_1 = 377.2$. At $t = 0.43 t_A$, the excess of the magnetic field maximum over α almost disappears. The temperature increases by a factor of 447.5, and the internal energy density, $\rho\varepsilon = P/(\gamma - 1)$, by a factor of 1265.7. Note that $\zeta - 1$ is only 1.7 at that time. Due to the enormous increase of the temperature, $\beta_2 = 1.58$. Finally, at $t = 1.00 t_A$, the magnetic maximum is well below α, and the temperature has been diffused due to the finite thermal conductivity. Nevertheless, $T_2/T_1 = 319.0$, $P_2/P_1 = 867.0$, and $\beta_2 = 1.29$.

Consider now $B_x \neq 0$, and suppose that initially we have only an incompressible motion, which cannot (at least initially) lead to compressive heating; at $t=0$, $v_z \neq 0$ while $v_x = 0$, $\rho = \text{const}$, and $B_z = B_{z1} + b_z$ [where $b_z(x, t=0) = 0$]. Then in the linear approximation, two waves—slow and fast—are excited, and the density fluctuations corresponding to the slow mode would be given by

$$\frac{\rho'}{\rho_1} \sim \frac{v_z(x,t=0)}{c_A} \frac{B_z}{B} \frac{c_A}{s}, \tag{2}$$

where s is the sound speed. If both $v_z(x,t=0)/c_A$ and B_z/B are not small parameters, then $\rho'/\rho_1 \sim 1/\beta^{1/2} \gg 1$, that is, the motion becomes highly compressible. Of course, the linear theory is not applicable anymore. For strong perturbations it is easy to construct the Riemann solution, e.g., $dv_x/d\rho = v_{sl}/\rho$, $dv_z/d\rho = -B_x dB_z/d\rho (4\pi\rho v_{sl})^{-1} = v_{sl} B_z / B_x$, where $v_{sl} = s B_x / B$ (viz., [2,5]). We then obtain

$$\rho = \rho_1 \left(1 - \frac{B_z^2 - B_{z1}^2 + B_x^2 \ln\{B_z^2/B_{z1}^2\}}{8\pi P(\rho_1)} \right)^{1/\gamma}. \tag{3}$$

It is clear from Eq. (3) that a modest decrease of B_z^2 corresponds to a substantial increase in ρ. For example, if $\delta B_z \sim B_z$, then $\rho/\rho_1 \sim \beta^{-1/\gamma} \gg 1$. Although the magnitude of the perturbation is now not restricted, the initial state is taken such that the Riemann solution evolves, and therefore Eq. (3) does not necessarily reflect what happens with arbitrary initial conditions [2,3]. The numerical simulations show that, if one starts with a *purely incompressible motion* at $t=0$, $v_z(x) \neq 0$, while $\rho(x) = \text{const}$, $v_x(x) = 0$, $B_z(x) = \text{const}$, $\beta(x) = \text{const} = 0.005$, then a perturbation of B_z starts to develop, leading to a compressible motion $v_x \neq 0$, which in turn generates a substantial density perturbation (Fig. 2). In this case, $\max(\rho)/\rho_1$ reaches 11.2, well above $\alpha(=4)$. As noted, this density increase was preceded by a chain of events, and this is the reason why the maximal value of ρ is reached only

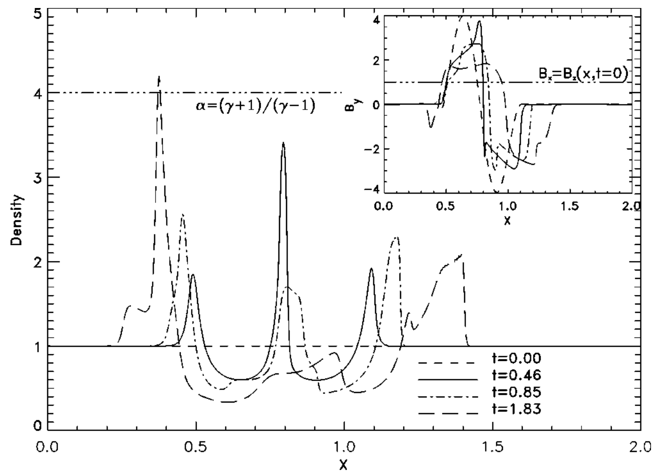

FIG. 3. Variation of the density excitation by a strong Alfvén perturbation. The inset shows the evolution of an Alfvénic fluctuation (B_y component) into an intermediate shock, which eventually diffuses. The initial B_z component, and B_x (which does not change), are also shown.

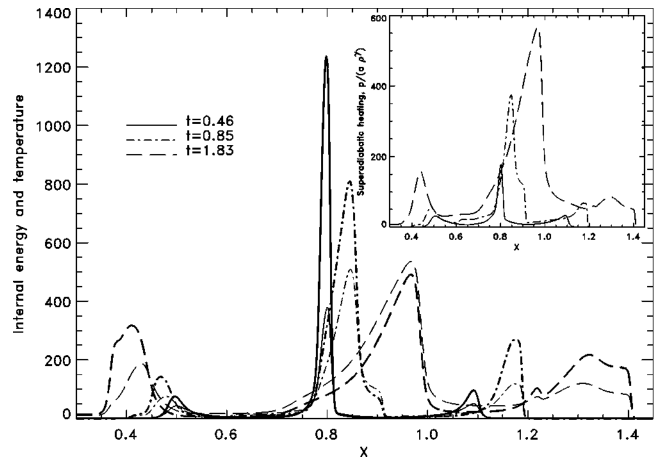

FIG. 4. Evolution of the internal energy density (thick lines) and of the temperature due to a strong initial Alfvénic perturbation. The inset illustrates the dynamics of superadiabatic heating.

at $t=2.07t_A$. However, once both the magnetic perturbation and v_x have grown sufficiently, the density perturbation rapidly increases (cf. Fig. 2): It takes only a fraction of one Alfvén time to reach the maximum. A sharp increase in ρ is, of course, accompanied by increasing pressure, so that β reaches ~ 0.31, substantially larger than at the beginning.

Consider now the evolution of a moderately strong Alfvén perturbation. Thus, at $t=0$ let $\rho=\text{const}$ $(\equiv 1)$, $B_x=\text{const}$, $B_y=B_y(x)$, assumed to be a function with finite support in space (i.e., $B_y\equiv 0$ outside some region; see Fig. 3), $v_y=-B_y/\sqrt{4\pi\rho}$, $B_z=\text{const}$, $v_x=v_z=0$, and $p=a=\text{const}$ $[\equiv a\rho(x,t=0)^\gamma]$, where the constant a is chosen in such a way that $\beta_1=p/(B_\perp^2/8\pi)=0.01$, where $B_y\equiv 0$. This Alfvén perturbation is purely incompressible. There is an exact solution which maintains this form indefinitely, always remaining incompressible,

$$B_\perp^2=B_y(x,t)^2+B_z(x,t)^2=\text{const} \tag{4}$$

[2,3,5]. However, an arbitrary perturbation with finite support cannot always evolve into this state. It follows from Eq. (4) that $B_z(x,t)^2=B_{z1}^2-B_y(x,t)^2$, but the RHS of this expression cannot be positive if B_y^2 is big enough, as in Fig. 3. There are other limitations for the intermediate shock waves due to the Hugoniot adiabat (see [11]). The two other modes—the fast and the slow modes—involve compression, and therefore they are expected to form shocks, thus efficiently converting the energy into heat [2–4]. For this reason, we expect that the strong perturbation under consideration would generate these two modes, at least until it gets rid of the ‘‘extra’’ amplitude of B_y, so that the remaining B_y can satisfy Eq. (4). On the other hand, a high-pressure plasma resists compression, and therefore we consider $\beta \ll 1$. As seen from Fig. 3, there are three places where the density is increasing, namely where the B_y component goes to zero. We start our discussion with the middle point.

As early as $t=0.46t_A$, the density gradient is substantially increased in the middle. This occurs because magnetic fields of opposite polarities (of the B_y component), with corresponding magnetic pressure, are not balanced by gas pressure, and therefore the plasma collapses to a place where $B_y=0$, quite similar to the formation of a current sheet in low-β plasmas (viz., [1]). Of course, the pressure grows due to this strong compression; in addition, however, there is superadiabatic heating [as measured by the ratio $p/(a\rho^\gamma)$] due to the sharp gradient of the B_y component (see the inset of Fig. 3), essentially heating in a current sheet. Equivalently, the heating corresponds to the after-shock energy increase, according to Eq. (1). The point here is that an intermediate shock is formed in this location. Indeed, the solid line in the inset corresponds to a quite typical profile of B_y for this shock; it moves to the right, and gradually dissipates. As seen from Fig. 4, the internal energy has grown dramatically by $t=0.46t_A$, in part because of adiabatic compression. Nevertheless, the temperature has increased 379.69-fold, and the maximal value of the superadiabatic heating $p/(a\rho^\gamma)$ is 178.11. However, the shock is only just formed, and superadiabatic heating now increases; as seen from the inset, at $t=0.85t_A$, i.e., only $0.39t_A$ later, the superadiabatic heating rate has reached a value of 374.64, that is, doubled. The temperature is also rapidly increasing; for example, at $t=0.35t_A$ it reaches the value of 245.93, and $0.35t_A$ later, it has also roughly doubled. It continues to grow, reaching the value of 548.68 at $t=1.42t_A$. At this point, the heating is so intense that the local $\beta=4.01$ (having started initially at 0.01).

The other two places where the shock waves are formed are at the edges of the excited region, at places where $B_y\to 0$. As seen from Fig. 3, the left shock is more intense, the asymmetry attributable to the fact that the perturbation as a whole is moving to the right (due to initial v_y). Apparently, the left shock corresponds to a slow shock [and indeed, there is a depletion of B_z^2 in this location, as in Eq. (3)], and therefore the density perturbation is relatively high here, even exceeding α (in related simulations with smaller initial B_z the density reaches a value of 4.96). The right shock corresponds to a fast mode. Consider the left perturbation in more detail. As for the slow mode considered above (Fig. 2), significant changes in local conditions occur only after a delay of a time $\sim t_A$. However, once the density perturbation has been formed, the energy release becomes rather efficient. Thus, the temperature at the left front is 75.47 at $t=0.85t_A$ (shown in Fig. 4) and is 162.68 at $t=1.42t_A$, i.e., it has doubled on a time scale of $0.57t_A$. As seen from the inset, the heating is mainly superadiabatic.

In conclusion, we have shown that a moderate ($\delta B/B \sim 1$) magnetic perturbation may result in efficient and fast energy release. This is especially true for perturbations exceeding the Hugoniot limit, and, as we demonstrated, this is easily achieved in a low-β plasma. As an example, consider the surface layers of a neutron star. It is believed [12] that the surface is covered by a strongly collisional plasma ocean $\sim$10 m deep: At the depth $d=13$ cm, $\rho\sim 10^4$ g cm^{-3}, and the pressure is $P\sim 10^{19}$ dyn cm^{-2}. We take $B=10^{12}$ G; the corresponding Alfvén velocity is 2.82×10^9 cm s^{-1} (justifying our nonrelativistic approach). The plasma $\beta\sim 0.000\,25$, so the theory described here applies. Suppose that compression is finite, e.g., let $\zeta=2$ in Eq. (1). Then, neglecting P_1, we obtain $P_2=0.5B^2/(8\pi)$ or $\rho_2\varepsilon_2 =(3/2)P_2=(3/4)B^2/(8\pi)$. Thus, the internal energy per unit volume, $\rho_2\varepsilon_2$, is now comparable to the magnetic energy density, and is $1/\beta$ larger than initially (i.e., 4000 times larger). Suppose that the field is excited on the scale $\sim 3.1\times 10^5$ cm; then the perturbation will pass through this distance in 1 millisecond. As the magnetic Mach number is of order of unity, it would take approximately the same time for the shock to form. Thus, the total energy deposited to the plasma can be estimated as $\int \rho_2\varepsilon_2 d\mathbf{x} \approx 8.89\times 10^{38}$ ergs, this energy release being comparable to that of x-ray bursts [13].

The dramatic evolutionary behavior of the magnetic field and temperature can be seen in the animated pictures at http://flash.uchicago.edu/~samuel/shock. The numerical code used in this paper is based on that created by Z. Mikić and described in [8]. We also thank L.I. Rudakov, V.D. Shapiro, A.M. Dykhne, B.C. Low, R.A. Chevalier, Z. Mikić, E.N. Parker, E. Brown, and L.P. Kadanoff for helpful discussions. This work was supported in part by the DOE ASCI/ Alliances program at the University of Chicago, Grant No. B341495.

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